8. Evaluate $\iint_R e^{-x-y}dxdy$, where R is the region in the first quadrant in which $x+y \le 1$.
Solution:

$$\begin{aligned}\int_{x=0}^{1}\int_{y=0}^{1-x} e^{-x}e^{-y}dydx &= \int_0^1 e^{-x}(1-e^{-(1-x)})dx \\ &= \int_0^1 (e^{-x}-e^{-1})dx = 1-2e^{-1}.\end{aligned}$$

9. Evaluate $\iint_R e^{-x-2y}dxdy$, where R is the region in the first quadrant in which $x \le y$
Solution:

$$\int_{x=0}^{\infty}\int_{y=x}^{\infty} e^{-x-2y}dydx = \int_0^{\infty}\frac{1}{2}e^{-3x}dx = \frac{1}{6}$$

10. Evaluate $\iint_R (x^2+y^2)dxdy$, where R is the region $0 \le x \le y \le L$
Solution:

$$\begin{aligned}\int_{y=0}^{L}\int_{x=0}^{y}(x^2+y^2)dydx &= \int_{y=0}^{L}\left(\frac{1}{3}x^3+y^2x\right)\Big|_{x=0}^{y}dydx \\ &= \int_0^L \frac{4}{3}y^3dy = \frac{L^4}{3}.\end{aligned}$$

11. Evaluate $\iint_R (x-y+1)dxdy$, where R is the region inside the unit square in which $x+y \ge 0.5$.
Solution:

$$\begin{aligned}&\int_{x=0}^{0.5}\int_{y=0.5-x}^{1}(x-y+1)dydx + \int_{x=0.5}^{1}\int_{y=0}^{1}(x-y+1)dydx \\ &= \int_{x=0}^{0.5}\left(xy-\frac{1}{2}y^2+y\right)\Big|_{y=0.5-x}^{1}dx + \int_{x=0.5}^{1}\left(xy-\frac{1}{2}y^2+y\right)\Big|_{y=0}^{1}dx \\ &= \int_0^{0.5}\left(x(1-\frac{1}{2}+x)-\frac{1}{2}(1-(\frac{1}{2}-x)^2)+(1-\frac{1}{2}+x)\right)dx \\ &\quad + \int_{0.5}^{1}\left(x+\frac{1}{2}\right)dx \\ &= \int_0^{0.5}\left(\frac{1}{8}+x+\frac{3}{2}x^2\right)dx + \left(\frac{1}{2}x^2+\frac{1}{2}x\right)\Big|_{0.5}^{1} \\ &= \frac{1}{2}\cdot\frac{1}{8}+\frac{1}{2\cdot 2^2}+\frac{1}{3\cdot 2^3}\cdot\frac{3}{2}+\frac{3}{8}+\frac{1}{4}=\frac{7}{8}\end{aligned}$$

12. Evaluate $\int_0^1\int_0^1 x\max(x,y)dydx$.
Solution:

$$\begin{aligned}\int_{x=0}^{1}\int_{y=0}^{x}x^2dydx + \int_{x=0}^{1}\int_{y=x}^{1}xydydx &= \int_0^1\left(x^3+x\frac{1-x^2}{2}\right)dx \\ &= \frac{1}{4}+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{3}{8}.\end{aligned}$$

Once the function is saved, it is used in the Command Window to solve the two cases. For case *a*) $A_0 = 67$, $A(t_1) = 79$, $t_1 = 6$, and $t = 20$:

```
>> expGD(67,79,6,20)
ans =
        116.03
```

Estimation of the population in the year 2000.

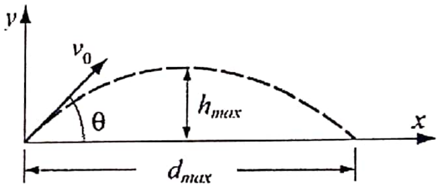
For case *b*) $A_0 = 7$, $A(t_1) = 3.5$ (since t_1 corresponds to the half-life, which is the time required for the material to decay to half of its initial quantity), $t_1 = 5.8$, and $t = 30$.

```
>> expGD(7,3.5,5.8,30)
ans =
          0.19
```

The amount of material after 30 years.

Sample Problem 7-6: Motion of a projectile

Create a function file that calculates the trajectory of a projectile. The inputs to the function are the initial velocity and the angle at which the projectile is fired. The outputs from the function are the maximum height and distance. In addition, the function generates a plot of the trajectory. Use the function to calculate the trajectory of a projectile that is fired at a velocity of 230 m/s at an angle of 39°.



Solution

The motion of a projectile can be analyzed by considering the horizontal and vertical components. The initial velocity v_0 can be resolved into horizontal and vertical components

$$v_{0x} = v_0\cos(\theta) \quad \text{and} \quad v_{0y} = v_0\sin(\theta)$$

In the vertical direction the velocity and position of the projectile are given by:

$$v_y = v_{0y} - gt \quad \text{and} \quad y = v_{0y}t - \frac{1}{2}gt^2$$

The time it takes the projectile to reach the highest point ($v_y = 0$) and the corresponding height are given by:

$$t_{hmax} = \frac{v_{0y}}{g} \quad \text{and} \quad h_{max} = \frac{v_{0y}^2}{2g}$$

The total flying time is twice the time it takes the projectile to reach the highest point, $t_{tot} = 2t_{hmax}$. In the horizontal direction the velocity is constant, and the position of the projectile is given by:

$$x = v_{0x}t$$

When the script file is executed, the following (the values of the variables `B`, `t`, `years`, and `months`) is displayed in the Command Window:

```
>> format short g
B =

       20011

t =

      16.374

years =
    16

months =
     5
```

The values of the variables `B`, `t`, `years`, and `months` are displayed (since a semicolon was not typed at the end of any of the commands that calculate the values).

1.10 PROBLEMS

The following problems can be solved by writing commands in the Command Window, or by writing a program in a script file and then executing the file.

1. Calculate:

(*a*) $\frac{(14.8^2 + 6.5^2)}{3.8^2} + \frac{55}{\sqrt{2} + 14}$ (*b*) $(-3.5)^3 + \frac{e^6}{\ln 524} + 206^{1/3}$

2. Calculate:

(*a*) $\frac{16.5^2(8.4 - \sqrt{70})}{4.3^2 - 17.3}$ (*b*) $\frac{5.2^3 - 6.4^2 + 3}{1.6^8 - 2} + \left(\frac{13.3}{5}\right)^{1.5}$

3. Calculate:

(*a*) $15\left(\frac{\sqrt{10} + 3.7^2}{\log_{10}(1365) + 1.9}\right)$ (*b*) $\frac{2.5^3\left(16 - \frac{216}{22}\right)}{1.7^4 + 14} + \sqrt[4]{2050}$

4. Calculate:

(*a*) $\frac{2.3^2 \cdot 1.7}{\sqrt{(1 - 0.8^2)^2 + (2 - \sqrt{0.87})^2}}$ (*b*) $2.34 + \frac{1}{2}2.7(5.9^2 - 2.4^2) + 9.8\ln 51$

5. Calculate:

(a) $\dfrac{\sin\left(\frac{7\pi}{9}\right)}{\cos^2\left(\frac{5}{7}\pi\right)} + \frac{1}{7}\tan\left(\frac{5}{12}\pi\right)$ (b) $\dfrac{\tan 64^\circ}{\cos^2 14^\circ} - \dfrac{3\sin 80^\circ}{\sqrt[3]{0.9}} + \dfrac{\cos 55^\circ}{\sin 11^\circ}$

6. Define the variable x as $x = 2.34$, then evaluate:

(a) $2x^4 - 6x^3 + 14.8x^2 + 9.1$ (b) $\dfrac{e^{2x}}{\sqrt{14 + x^2 - x}}$

7. Define the variable t as $t = 6.8$, then evaluate:

(a) $\ln(|t^2 - t^3|)$ (b) $\dfrac{75}{2t}\cos(0.8t - 3)$

8. Define the variables x and y as $x = 8.3$ and $y = 2.4$, then evaluate:

(a) $x^2 + y^2 - \dfrac{x^2}{y^2}$ (b) $\sqrt{xy} - \sqrt{x+y} + \left(\dfrac{x-y}{x-2y}\right)^2 - \sqrt{\dfrac{x}{y}}$

9. Define the variables a, b, c, and d as:

$a = 13$, $b = 4.2$, $c = (4b)/a$, and $d = \dfrac{abc}{a+b+c}$, then evaluate:

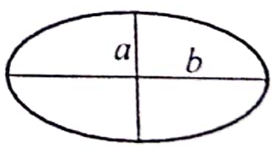
(a) $a\dfrac{b}{c+d} + \dfrac{da}{cb} - (a - b^2)(c + d)$ (b) $\dfrac{\sqrt{a^2 + b^2}}{(d - c)} + \ln(|b - a + c - d|)$

10. A cube has a side of 18 cm.

(a) Determine the radius of a sphere that has the same surface area as the cube.

(b) Determine the radius of a sphere that has the same volume as the cube.

11. The perimeter P of an ellipse with semi-minor axes a and b is given approximately by: $P = 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$.



(a) Determine the perimeter of an ellipse with $a = 9$ in. and $b = 3$ in.

(b) An ellipse with $b = 2a$ has a perimeter of $P = 20$ cm. Determine a and b.

12. Two trigonometric identities are given by:

(a) $\sin 4x = 4\sin x\cos x - 8\sin^3 x\cos x$ (b) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = \frac{\pi}{9}$.

13. Two trigonometric identities are given by:

(*a*) $\tan 4x = \frac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$ (*b*) $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = 12°$.

14. Define two variables: $alpha = 5\pi/8$, and $beta = \pi/8$. Using these variables, show that the following trigonometric identity is correct by calculating the values of the left and right sides of the equation.

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

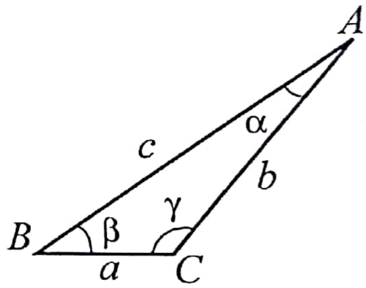
15. Given: $\int \cos^2(ax)dx = \frac{1}{2}x - \frac{\sin 2ax}{4a}$. Use MATLAB to calculate the following definite integral: $\int_{\frac{\pi}{9}}^{\frac{3\pi}{5}} \cos^2(0.5x)dx$.

16. In the triangle shown $a = 9$ cm, $b = 18$ cm, and $c = 25$ cm. Define *a*, *b*, and *c* as variables, and then:

(*a*) Calculate the angle α (in degrees) by substituting the variables in the Law of Cosines. (Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

(*b*) Calculate the angles β and γ (in degrees) using the Law of Sines.

(*c*) Check that the sum of the angles is 180°.



17. In the triangle shown $a = 5$ in., $b = 7$ in., and $\gamma = 25°$. Define *a*, *b*, and γ as variables, and then:

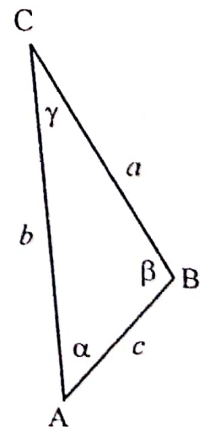
(*a*) Calculate the length of *c* by substituting the variables in the Law of Cosines.

(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

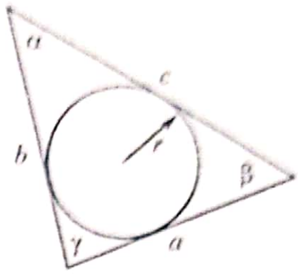
(*b*) Calculate the angles α and β (in degrees) using the Law of Sines.

(*c*) Verify the Law of Tangents by substituting the results from part (*b*) into the right and left sides of the equation.

(Law of Tangents: $\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha - \beta)\right]}{\tan\left[\frac{1}{2}(\alpha + \beta)\right]}$



18. For the triangle shown, $a = 200$ mm, $b = 250$ mm, and $c = 300$ mm. Define a, b, and c as variables, and then:



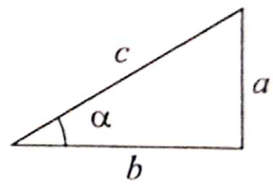
(*a*) Calculate the angle γ (in degrees) by substituting the variables in the Law of Cosines.

(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

(*b*) Calculate the radius r of the circle inscribed in the triangle using the formula $r = \frac{1}{2}(a + b - c)\tan\left(\frac{1}{2}\gamma\right)$.

(*c*) Calculate the radius r of the circle inscribed in the triangle using the formula $r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$, where $s = \frac{1}{2}(a + b + c)$.

19. In the right triangle shown $a = 16$ cm and $c = 50$ cm. Define a and c as variables, and then:



(*a*) Using the Pythagorean Theorem, calculate b by typing one line in the Command Window.

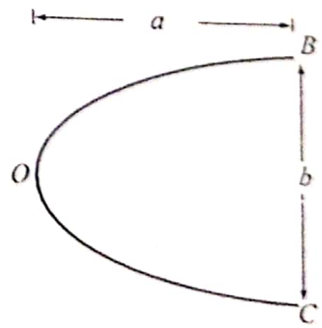
(*b*) Using b from part (*a*) and the `acosd` function, calculate the angle α in degrees by typing one line in the Command Window.

20. The distance d from a point (x_0, y_0, z_0) to a plane $Ax + By + Cz + D = 0$ is given by:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Determine the distance of the point $(8, 3, -10)$ from the plane $2x + 23y + 13z - 24 = 0$. First define the variables A, B, C, D, x_0, y_0, and z_0, and then calculate d. (Use the `abs` and `sqrt` functions.)

21. The arc length s of the parabolic segment BOC is given by:

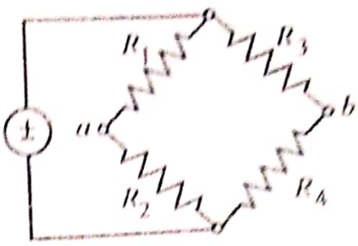


$$s = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$

Calculate the arc length of a parabola with $a = 12$ in. and $b = 8$ in.

22. Oranges are packed such that 52 are placed in each box. Determine how many boxes are needed to pack 4,000 oranges. Use MATLAB built-in function `ceil`.

23. The voltage difference V_{ab} between points a and b in the Wheatstone bridge circuit is:



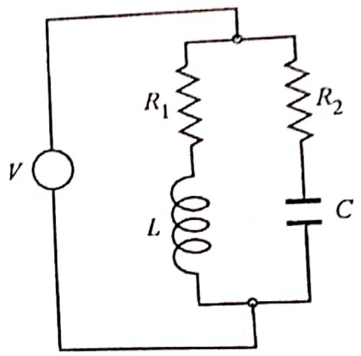
$$V_{ab} = V\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right)$$

Calculate the voltage difference when $V = 12$ volts, $R_1 = 120$ ohms, $R_2 = 100$ ohms, $R_3 = 220$ ohms, and $R_4 = 120$ ohms.

24. The prices of an oak tree and a pine tree are \$54.95 and \$39.95, respectively. Assign the prices to variables named oak and pine, change the display format to `bank`, and calculate the following by typing one command:

(*a*) The total cost of 16 oak trees and 20 pine trees.
(*b*) The same as part (*a*), and add 6.25% sale tax.
(c) The same as part (*b*) and round the total cost to the nearest dollar.

25. The resonant frequency f (in Hz) for the circuit shown is given by:



$$f = \frac{1}{2\pi}\sqrt{LC\frac{R_1^2C - L}{R_2^2C - L}}$$

Calculate the resonant frequency when $L = 0.2$ henrys, $R_1 = 1500$ ohms, $R_2 = 1500$ ohms, and $C = 2 \times 10^{-6}$ farads.

26. The number of combinations $C_{n,r}$ of taking r objects out of n objects is given by:

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

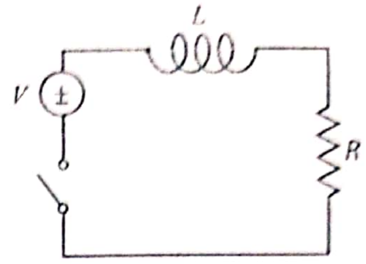
A deck of poker cards has 52 different cards. Determine how many different combinations are possible for selecting 5 cards from the deck. (Use the built-in function `factorial`.)

27. The formula for changing the base of a logarithm is:

$$\log_a N = \frac{\log_b N}{\log_b a}$$

(*a*) Use MATLAB's function `log(x)` to calculate $\log_4 0.085$.
(*b*) Use MATLAB's function `log10(x)` to calculate $\log_6 1500$.

28. The current I (in amps) t seconds after closing the switch in the circuit shown is:



$$I = \frac{V}{R}(1 - e^{-(R/L)t})$$

Given $V = 120$ volts, $R = 240$ ohms, and $L = 0.5$ henrys, calculate the current 0.003 seconds after the switch is closed.

29. Radioactive decay of carbon-14 is used for estimating the age of organic material. The decay is modeled with the exponential function $f(t) = f(0)e^{kt}$, where t is time, $f(0)$ is the amount of material at $t = 0$, $f(t)$ is the amount of material at time t, and k is a constant. Carbon-14 has a half-life of approximately 5,730 years. A sample of paper taken from the Dead Sea Scrolls shows that 78.8% of the initial ($t = 0$) carbon-14 is present. Determine the estimated age of the scrolls. Solve the problem by writing a program in a script file. The program first determines the constant k, then calculates t for $f(t) = 0.788f(0)$, and finally rounds the answer to the nearest year.

30. Fractions can be added by using the smallest common denominator. For example, the smallest common denominator of 1/4 and 1/10 is 20. Use the MATLAB Help Window to find a MATLAB built-in function that determines the least common multiple of two numbers. Then use the function to show that the least common multiple of:
(*a*) 6 and 26 is 78.
(*b*) 6 and 34 is 102.

31. The Moment Magnitude Scale (MMS), denoted M_W, which is used to measure the size of an earthquake, is given by:

$$M_W = \frac{2}{3}\log_{10} M_0 - 10.7$$

where M_0 is the magnitude of the seismic moment in dyne-cm (measure of the energy released during an earthquake). Determine how many times more energy was released from the earthquake in Sumatra, Indonesia ($M_W = 8.5$), in 2007 than the earthquake in San Francisco, California ($M_W = 7.9$), in 1906.

32. According to special relativity, a rod of length L moving at velocity v will shorten by an amount δ, given by:

$$\delta = L\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

where c is the speed of light (about 300×10^6 m/s). Calculate how much a rod 2 meter long will contract when traveling at 5,000 m/s.

33. The monthly payment M of a loan amount P for y years and interest rate r can be calculated by the formula:

$$M = \frac{P(r/12)}{1-(1+r/12)^{-12y}}$$

(*a*) Calculate the monthly payment of a $85,000 loan for 15 years and interest rate of 5.75% ($r = 0.0575$). Define the variables P, r, and y and use them to calculate M.

(*b*) Calculate the total amount needed for paying back the loan.

34. The balance B of a savings account after t years when a principal P is invested at an annual interest rate r and the interest is compounded yearly is given by $B = P(1+r)^t$. If the interest is compounded continuously, the balance is given by $B = Pe^{rt}$. An amount of $40,000 is invested for 20 years in an account that pays 5.5% interest and the interest is compounded yearly. Use MATLAB to determine how many fewer days it will take to earn the same if the money is invested in an account where the interest is compounded continuously.

35. The temperature dependence of vapor pressure p can be estimated by the Anteing equation:

$$\ln(p) = A - \frac{B}{C+T}$$

where ln is the natural logarithm, p is in mm Hg, T is in kelvins, and A, B, and C are material constants. For toluene ($C_6H_5CH_3$) in the temperature range from 280 to 410 K the material constants are $A = 16.0137$, $B = 3096.52$, and $C = -53.67$. Calculate the vapor pressure of toluene at 315 and 405 K.

36. Sound level L_P in units of decibels (dB) is determined by:

$$L_P = 20\log_{10}\left(\frac{p}{p_0}\right)$$

where p is the sound pressure of the sound, and $p_0 = 20 \times 10^{-6}$ Pa is a reference sound pressure (the sound pressure when $L_P = 0$ dB).

(*a*) The sound pressure of a passing car is 80×10^{-2} Pa. Determine its sound level in decibels.

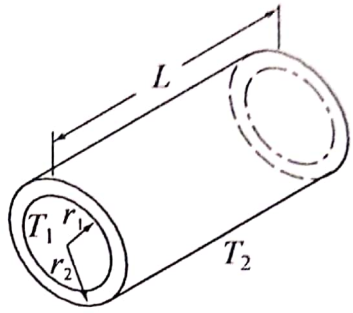
(*b*) The sound level of a jet engine is 110 decibels. By how many times is the sound pressure of the jet engine larger (louder) than the sound of the passing car?

37. Use the Help Window to find a display format that displays the output as a ratio of integers. For example, the number 3.125 will be displayed as 25/8. Change the display to this format and execute the following operations:

(a) $5/8 + 16/6$ (b) $1/3 - 11/13 + 2.7^2$

38. The steady-state heat conduction q from a cylindrical solid wall is determined by:

$$q = 2\pi Lk\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$



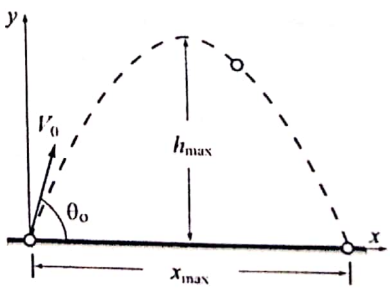
where k is the thermal conductivity. Calculate q for a copper tube (k = 401 Watts/°C/m) of length L = 300 cm with an outer radius of r_2 = 5 cm and an inner radius of r_1 = 3 cm. The external temperature is T_2 = 20° C and the internal temperature is T_1 = 100°C.

39. Stirling's approximation for large factorials is given by:

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Use the formula for calculating 20!. Compare the result with the true value obtained with MATLAB's built-in function `factorial` by calculating the error ($Error = (TrueVal - ApproxVal)/TrueVal$).

40. A projectile is launched at an angle θ and speed of V_0. The projectile's travel time t_{travel}, maximum travel distance x_{max}, and maximum height h_{max} are given by:

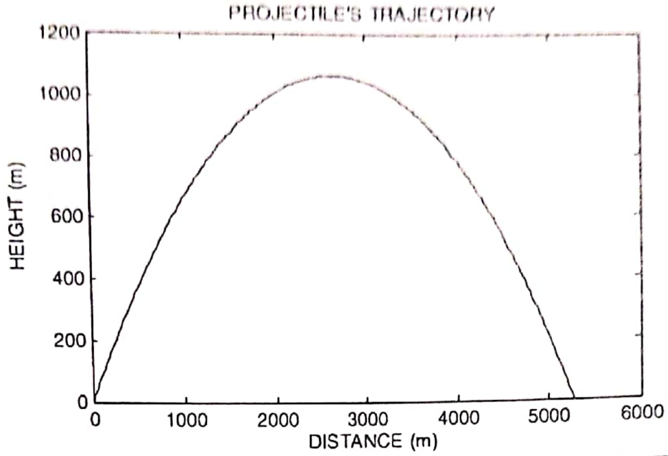


$$t_{travel} = 2\frac{V_0}{g}\sin\theta_0, \quad x_{max} = 2\frac{V_0^2}{g}\sin\theta_0\cos\theta_0,$$

$$h_{max} = 2\frac{V_0^2}{g}\sin^2\theta_0$$

Consider the case where V_0 = 600 ft/s and θ = 54°. Define V_0 and θ as MATLAB variables and calculate t_{travel}, x_{max}, and h_{max} (g = 32.2 ft/s^2).

In addition, the following figure is created in the Figure Window:



7.13 PROBLEMS

1. The fuel efficiency of an automobile is measured in mi/gal (miles per U.S. gallon) or in km/L (kilometers per liter). Write a MATLAB user-defined function that converts fuel efficiency values from km/L to mi/gal. For the function name and arguments use `mpg=kmlTOmpg(kml)`. The input argument `kml` is the efficiency in km/L, and the output argument `mpg` is the efficiency in mi/gal. Use the function in the Command Window to:
   (*a*) Determine the fuel efficiency in mi/gal of a car that consumes 9 km/L.
   (b) Determine the fuel efficiency in mi/gal of a car that consumes 14 km/L.

2. Write a user-defined MATLAB function for the following math function:
$$y(x) = -0.2x^4 + e^{-0.5x}x^3 + 7x^2$$
   The input to the function is x and the output is y. Write the function such that x can be a vector (use element-by-element operations).
   (*a*) Use the function to calculate $y(-2.5)$, and $y(3)$.
   (*b*) Use the function to make a plot of the function $y(x)$ for $-3 \le x \le 4$.

3. Write a user-defined MATLAB function, with two input and two output arguments, that determines the height in centimeters and mass in kilograms of a person from his height in inches and weight in pounds. For the function name and arguments use `[cm, kg] = STtoSI(in,lb)`. The input arguments are the height in inches and weight in pounds, and the output arguments are the height in centimeters and mass in kilograms. Use the function in the Command Window to:
   (*a*) Determine in SI units the height and mass of a 5 ft 8 in. person who weighs 175 lb.
   (b) Determine your own height and weight in SI units.

4. Write a user-defined MATLAB function that converts speed given in units of miles per hour to speed in units of meters per second. For the function name and arguments use `mps = mphTOmets(mph)`. The input argument is the speed in mi/h, and the output argument is the speed in m/s. Use the function to convert 55 mi/h to units of m/s.

5. Write a user-defined MATLAB function for the following math function:

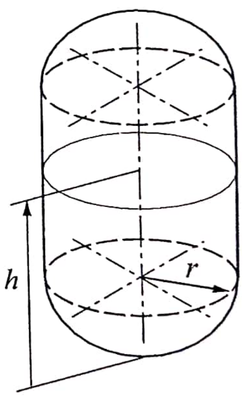
$$r(\theta) = 2\cos\theta\sin\theta\sin(\theta/4)$$

The input to the function is θ (in radians) and the output is r. Write the function such that θ can be a vector.
(*a*) Use the function to calculate $r(3\pi/4)$ and $r(7\pi/4)$.
(*b*) Use the function to plot (polar plot) $r(\theta)$ for $0 \le \theta \le 2\pi$.

6. Write a user-defined MATLAB function that determines the area of a triangle when the lengths of the sides are given. For the function name and arguments use `[Area] = triangle(a,b,c)`. Use the function to determine the areas of triangles with the following sides:
(*a*) $a = 3, b = 8, c = 10$. (*b*) $a = 7, b = 7, c = 5$.

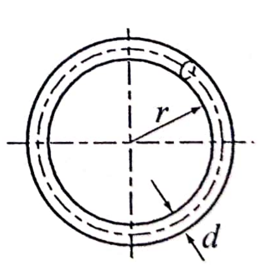
7. A cylindrical vertical fuel tank has hemispheric end caps as shown. The radius of the cylinder and the caps is r = 15 in., and the height of the cylindrical middle section is 40 in.

Write a user-defined function (for the function name and arguments use `V = Volfuel(h)`) that gives the volume of fuel in the tank (in gallons) as a function of the height h (measured from the bottom). Use the function to make a plot of the volume as a function of h for $0 \le h \le 70$ in.



8. The surface area S of a ring in shape of a torus with an inner radius r and a diameter d is given by:

$$S = \pi^2(2r + d)d$$



The ring is to be plated with a thin layer of coating. The weight of the coating W can be calculated approximately as $W = \gamma S t$, where γ is the specific weight of the coating material and t is its thickness. Write an anonymous function that calculates the weight of the coating. The function should have four input arguments, r, d, t, and γ. Use the anonymous function to calculate the weight of a gold coating (γ = 0.696 lb/in.^{3}) of a ring with r = 0.35 in., d = 0.12 in., and t = 0.002 in.

9. The monthly deposit into a savings account S needed to reach an investment goal B can be calculated by the formula

$$M = S\frac{\frac{r}{1200}}{\left(1+\frac{r}{1200}\right)^{12N}-1}$$

where M is the monthly deposit, S is the saving goal, N is the number of years, and r is the annual interest rate (%). Write a MATLAB user-defined function that calculates the monthly deposit into a savings account. For the function name and arguments use `M = invest(S,r,N)`. The input arguments are `S` (the investment goal), `r` (the annual interest rate, %), and `N` (duration of the savings in years). The output `M` is the amount of the monthly deposit. Use the function to calculate the monthly deposit for a 10-year investment if the investment goal is $25,000 and the annual interest rate is 4.25%.

10. The heat index, HI (in degrees F), is an apparent temperature. For temperatures higher than 80°F and humidity higher than 40% it is calculated by:

$$HI = C_1 + C_2T + C_3R + C_4TR + C_5T^2 + C_6R^2 + C_7T^2R + C_8TR^2 + C_9R^2T^2$$

where T is temperature in degrees F, R is the relative humidity in percent, $C_1 = -42.379$, $C_2 = 2.04901523$, $C_3 = 10.14333127$, $C_4 = -0.22475541$, $C_5 = -6.83783 \times 10^{-3}$, $C_6 = -5.481717 \times 10^{-2}$, $C_7 = 1.22874 \times 10^{-3}$, $C_8 = 8.5282 \times 10^{-4}$, and $C_9 = -1.99 \times 10^{-6}$. Write a user-defined function for calculating HI for given T and R. For the function name and arguments use `HI=HeatIn(T,R)`. The input arguments are `T` in °F and, `R` in %, and the output argument is `HI` in °F (rounded to the nearest integer). Use the function to determine the heat index for the following conditions:

(*a*) $T = 95$ °F, $R = 80$ %.

(*b*) $T = 100$ °F, $R = 100$ % (condition in a sauna).

11. The body fat percentage (BFP) of a person can be estimated by the formula

$$BFP = 1.2 \times BMI + 0.23 \times Age - 10.8 \times Gender - 0.54$$

where BMI is the body mass index, given by $BMI = 703\frac{W}{H^2}$, in which W is the weight in pounds and H is the height in inches, Age is the person's age, and $Gender = 1$ for a male and $Gender = 0$ for a female.

Write a MATLAB user-defined function that calculates the body fat percentage. For the function name and arguments use `BFP = BodyFat(w,h,age,gen)`. The input arguments are the weight, height, age, and gender (1 for male, 0 for female), respectively. The output argument is the BEF value. Use the function to calculate the body fat percentage of:

a) A 35-years-old, 6 ft 2 in. tall, 220 lb male.

b) A 22-years-old, 5 ft 7 in. tall, 135 lb female.

12. Write a user-defined function that calculates grade point average (GPA) on a scale of 0 to 4, where $A = 4$, $B = 3$, $C = 3$, $D = 1$, and $E = 0$. For the function name and arguments use `av = GPA(g,h)`. The input argument `g` is a vector whose elements are letter grades *A*, *B*, *C*, *D*, or *E* entered as strings. The input argument `h` is a vector with the corresponding credit hours. The output argument `av` is the calculated GPA. Use the function to calculate the GPA for a student with the following record:

Grade	B	A	C	E	A	B	D	B
Credit Hours	3	4	3	4	3	4	3	2

For this case the input arguments are:
`g=['BACEABDB']` and `h=[3 4 3 4 3 4 3 2]`.

13. The factorial $n!$ of a positive number (integer) is defined by $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$, where $0! = 1$. Write a user-defined function that calculates the factorial $n!$ of a number. For function name and arguments use `y=fact(x)`, where the input argument `x` is the number whose factorial is to be calculated, and the output argument `y` is the value $x!$. The function displays an error message if a negative or non-integer number is entered when the function is called. Use `fact` with the following numbers:
(*a*) 12! (*b*) 0! (*c*) –7! (*d*) 6.7!

14. Write a user-defined MATLAB function that determines the vector connecting two points (*A* and *B*). For the function name and arguments use `V=vector(A,B)`. The input arguments to the function are vectors `A` and `B`, each with the Cartesian coordinates of points *A* and *B*. The output `V` is the vector from point *A* to point *B*. If points *A* and *B* have two coordinates each (they are in the *x y* plane), then `V` is a two-element vector. If points *A* and *B* have three coordinates each (general points in space), then `V` is a three-element vector. Use the function `vector` for determining the following vectors.
(*a*) The vector from point (0.5, 1.8) to point (–3, 16).
(*b*) The vector from point (–8.4, 3.5, –2.2) to point (5, –4.6, 15).

15. Write a user-defined MATLAB function that determines the dot product of two vectors. For the function name and arguments use `D=dotpro(u,v)`. The input arguments to the function are two vectors, which can be two- or three-dimensional. The output `D` is the result (a scalar). Use the function `dotpro` for determining the dot product of:
(*a*) Vectors $a = 3i + 11j$ and $b = 14i - 7.3j$.
(*b*) Vectors $c = -6i + 14.2j + 3k$ and $d = 6.3i - 8j - 5.6k$.